Positive solutions for singular elliptic problems

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We present our results regarding the existence of positive solutions of the following problem

$$\begin{cases} \Delta u(x) + f(x, u(x)) - b(x)(u(x))^{-\alpha}||\nabla u(x)||^{\beta} + g(x)x \cdot \nabla u(x) = 0, \\ \lim\limits_{||x||\to\infty} u(x) = 0, \end{cases} \tag{1}$$

where $n > 2$, $R > 1$, $||x|| := \sqrt{\sum_{i=1}^{n} x_i^2}$, $\Omega_R = \{x \in \mathbb{R}^n, ||x|| > R\}$, $0 < \alpha \leq \beta - \alpha$, which originates from the problem formulated in works of A. Constantin ([1]) without the singular part.

In the first part of the presentation, we consider a problem without a singular part and show the multiplicity of solutions. As our main tool we use the Noussair-Swanson theorem concerning the sub-supersolution ([3]).

In the second part, we show the existence of a classical solution to (1) using both sub-supersolution method ([2]) as well as the unbounded domain approximation method, described, among others in [3] and [4]. Here we use solutions from the first part as supersolutions of (1).

References

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